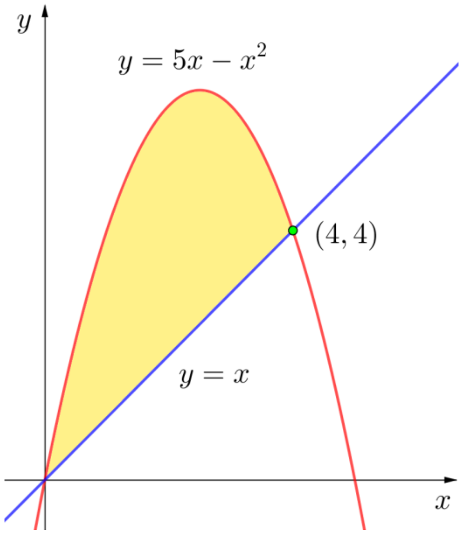
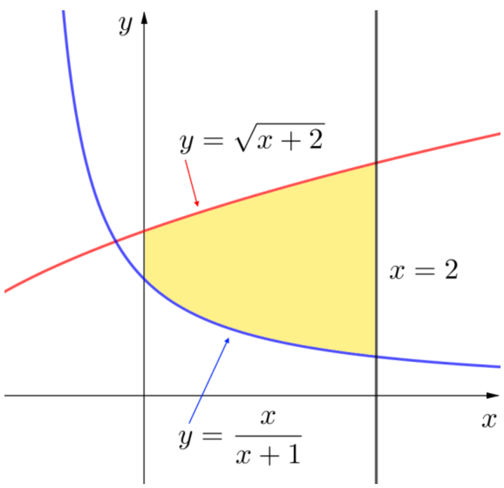
Find the area of the shaded region.

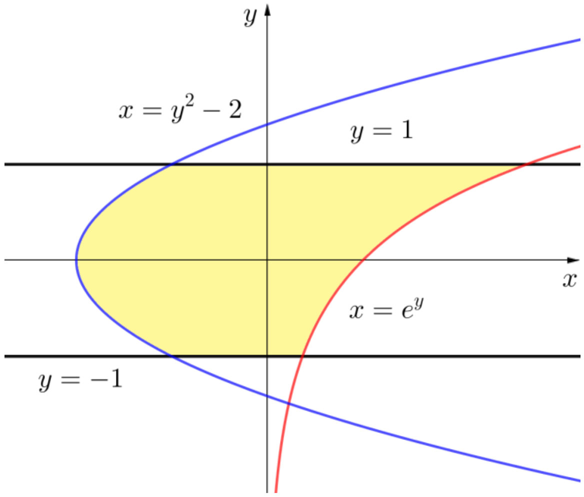
1) 

$\boxed{\dfrac{32}{3}}$

2) 

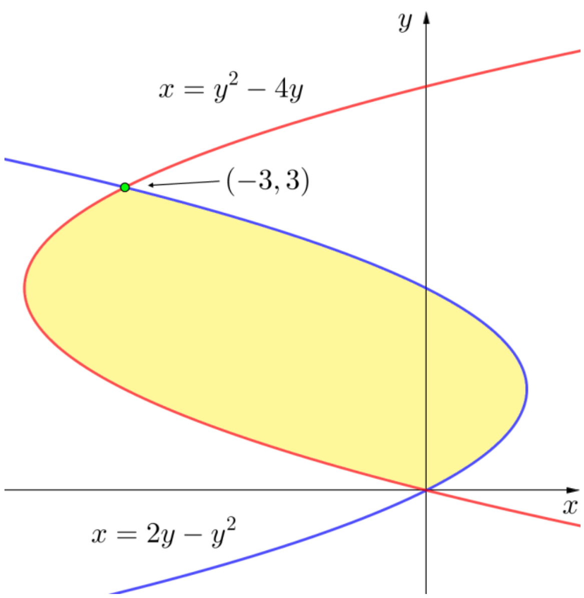
$\boxed{\dfrac{16}{3} - \ln 3 - \dfrac{4}{3}\sqrt{2}}$

3)



$$e - \frac{1}{e} + \frac{10}{3}$$

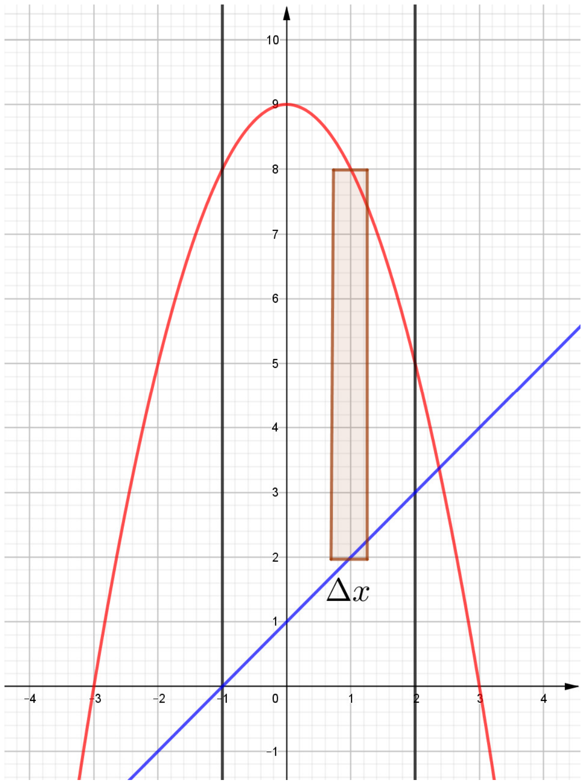
4)



$$9$$

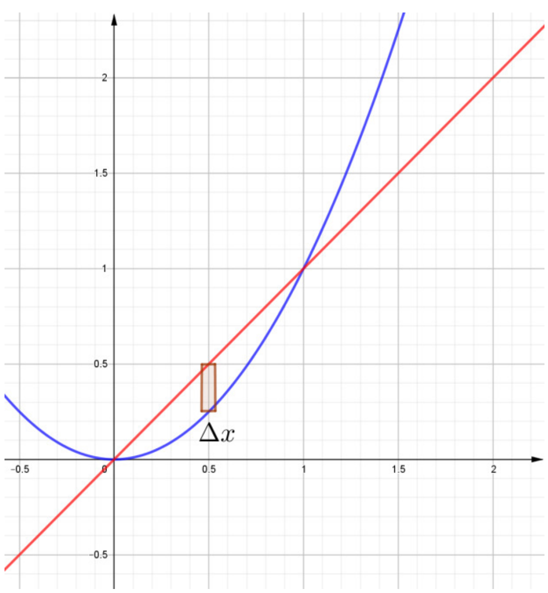
Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

5) $y = x+1, \quad y = 9 - x^2, \quad x = -1, \quad x = 2$



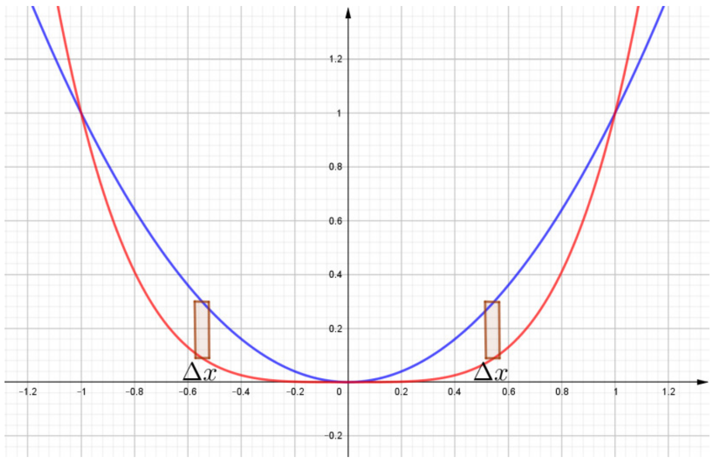
$\boxed{\dfrac{39}{2}}$

6) $y = x, \quad y = x^2$



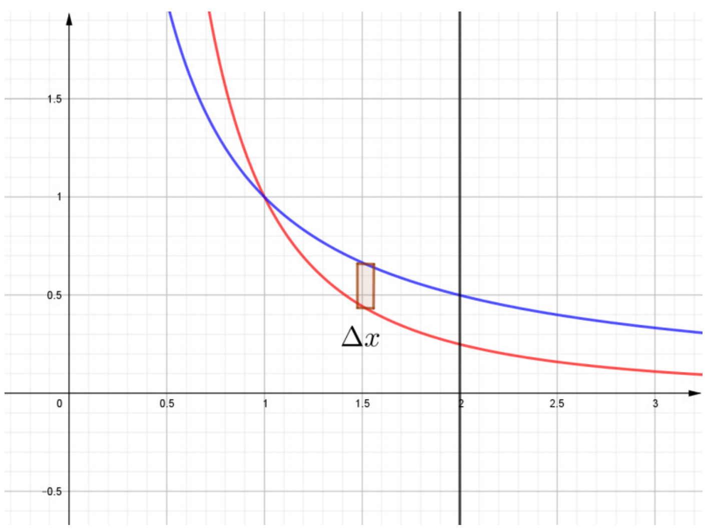
$\boxed{\dfrac{1}{6}}$

7) $y = x^2, \quad y = x^4$



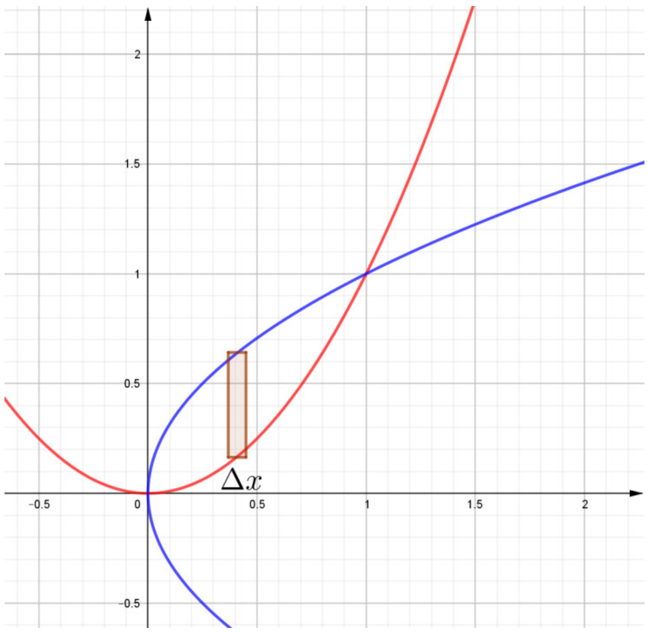
$\boxed{\dfrac{4}{15}}$

8) $y = \dfrac{1}{x}, \quad y = \dfrac{1}{x^2}, \quad x = 2$



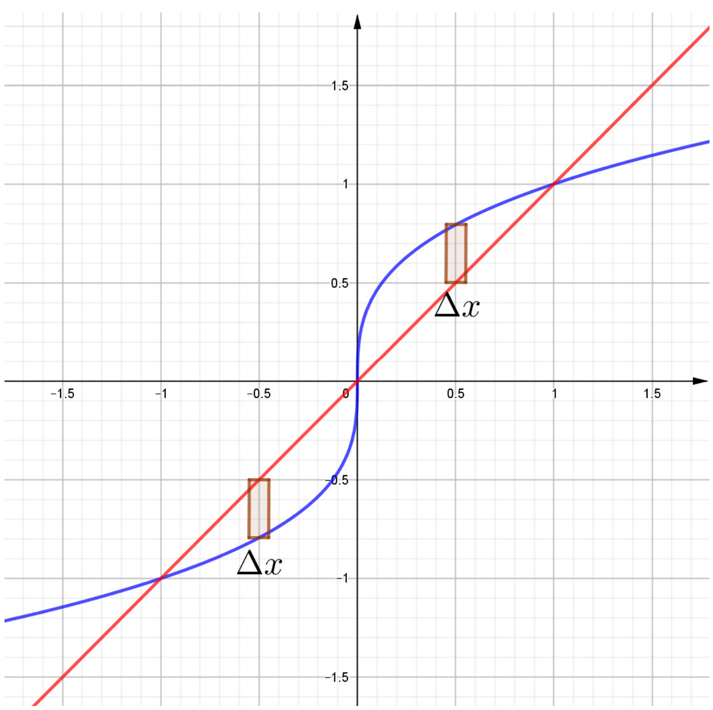
$\boxed{\ln 2 - \dfrac{1}{2}}$

9) $y = x^2, \quad y^2 = x$



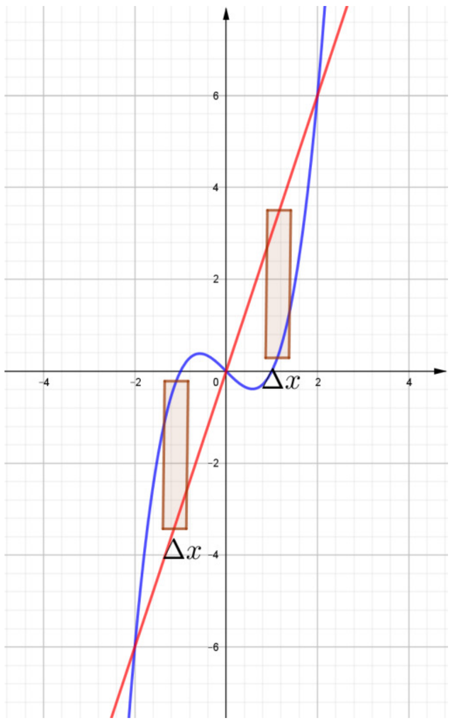
$\boxed{\frac{1}{3}}$

10) $y = x, \quad y = \sqrt[3]{x}$



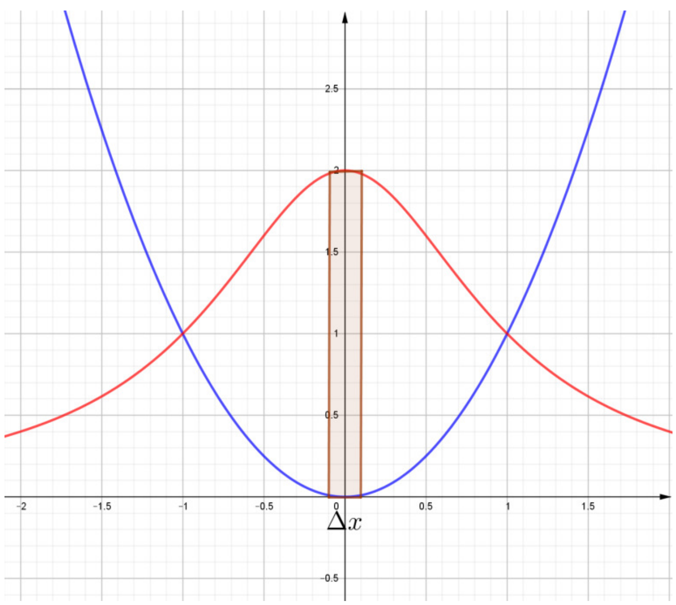
$\boxed{\frac{1}{2}}$

11) $y = x^3 - x, \quad y = 3x$



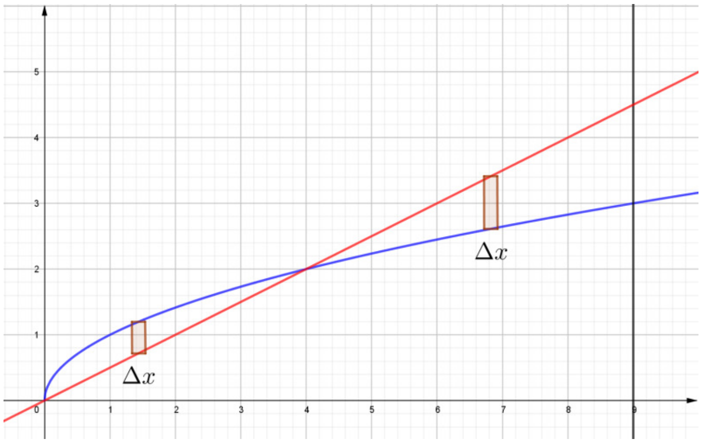
$\boxed{8}$

12) $y = x^2, \quad y = \dfrac{2}{x^2 + 1}$



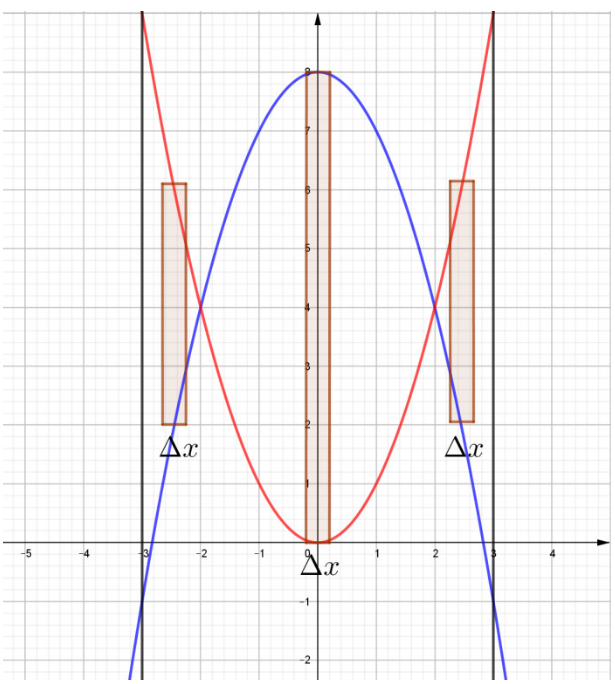
$\boxed{\pi - \dfrac{2}{3}}$

13) $y = \sqrt{x}, \quad y = \frac{1}{2}x, \quad x = 9$



$\boxed{\frac{59}{12}}$

14) $y = 8 - x^2, \quad y = x^2, \quad x = -3, \quad x = 3$

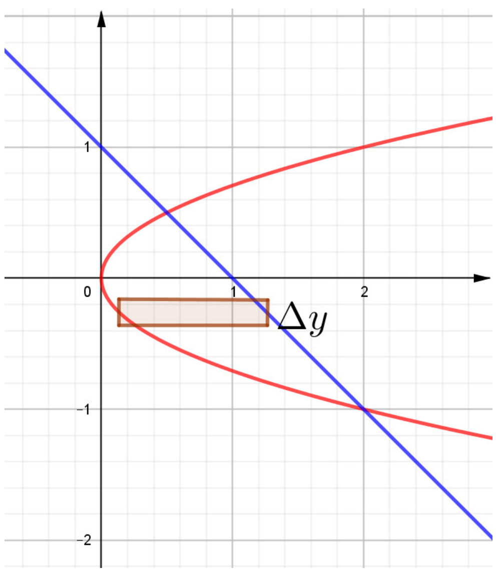


$\boxed{\frac{92}{3}}$

15) $y=\cos x, \quad y=\sin 2x, \quad x=0, \quad x=\dfrac{\pi}{2}$

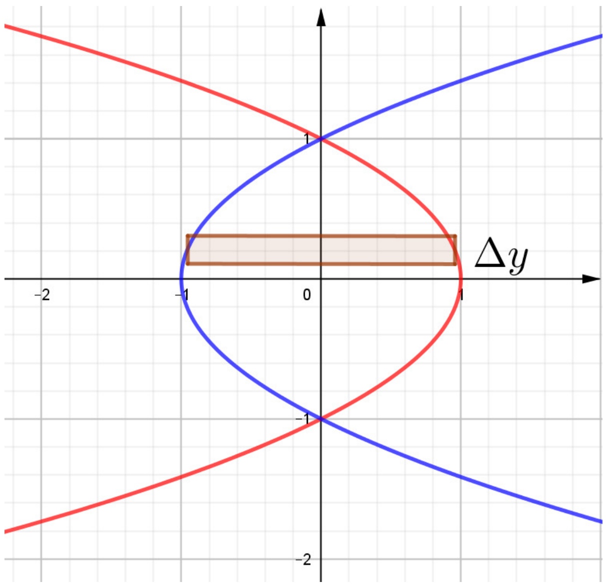
$\boxed{\dfrac{1}{2}}$

16) $x=2y^2, \quad x+y=1$



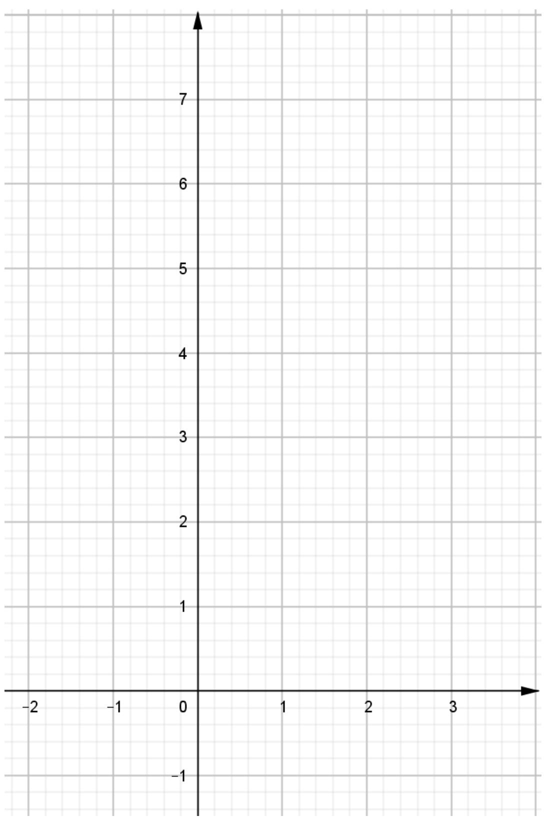
$\boxed{\dfrac{9}{8}}$

17) $x = 1 - y^2, \quad x = y^2 - 1$



$\boxed{\frac{8}{3}}$

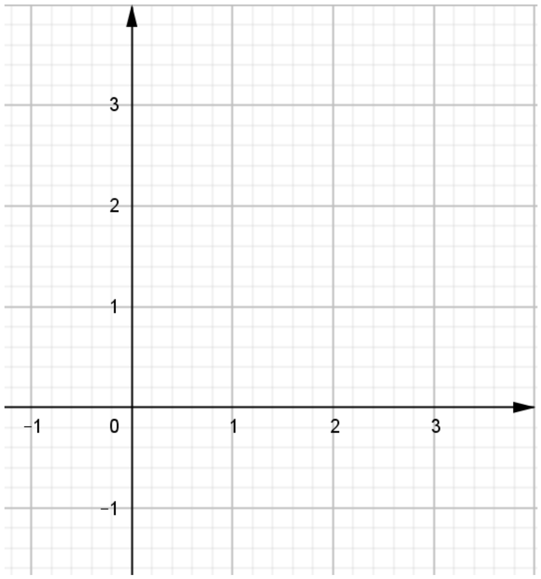
18) Use calculus to find the area of the triangle with the given vertices: $(0,0),\ (2,1),\ (-1,6)$



$\boxed{\frac{13}{2}}$

19) Use the Midpoint Rule with $n = 4$ to approximate the area of the region bounded by the given curves:

$$y = \sqrt[3]{16 - x^3}, \quad y = x, \quad x = 0$$



$\boxed{A \approx 2.8144}$

Use a graphing calculator to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.

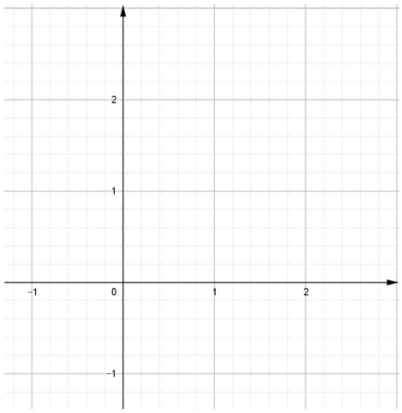
20) $y = x^2, \quad y = 2\cos x$ $\boxed{A \approx 2.70}$

21) $y = x\cos(x^2), \quad y = x^3$ $\boxed{A \approx 0.40}$

22) The curve with equation $y^2 = x^2(x+3)$ is called **Tschirnhausen's cubic**. If you graph this curve you will see that part of the curve forms a loop. Find the area enclosed by the loop.

$$\boxed{\frac{24}{5}\sqrt{3}}$$

23) Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1,1)$, and the x-axis.



$\boxed{\dfrac{1}{12}}$

24) Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.

$\boxed{b = 4^{\frac{2}{3}}}$

25) Find the values of c such that the area of the region enclosed by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576.

$\boxed{c = \pm 6}$